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PRELIMINARY REPORT: STUDY OF THE EFFECTS OF AIR POLLUTION AND WEATHER ON CARDIOVASCULAR MORTALITY IN CINCINNATI, 1952

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April 1955

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ROBERT A. TAFT SANITARY ENGINEERING CENTER
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Preliminary Report: Study of the Effects of Air Pollution and Weather on Cardiovascular Mortality in Cincinnati, 1952

I. Materials

This study is based on records of environmental conditions and cardiovascular mortality during 1952 in Cincinnati.

A. Weather Data:

Daily weather observations were obtained from published reports of the United States Weather Bureau. Specific weather variables examined in this study are included in Table I. The Weather Bureau maintains three observation posts in the Cincinnati area: one, the City Office, on the tenth floor of Carew Tower, a downtown office building, another at Abbe Observatory in the eastern part of the city, some seven miles from the downtown area, and a third at the Greater Cincinnati Airport in Boone County, Kentucky, about fifteen miles from downtown Cincinnati. Observations recorded at the City Office were preferred in order to obtain as representative a picture as possible of weather conditions affecting Cincinnati residents. Abbe Observatory provided data on percent of possible sunshine, not available from the City Office; Airport records were used to obtain information on relative humidity and barometric pressure.

B. Air Pollution Measurements:

The installation of a Hemeon automatic smoke filter at the site of the Bureau of Smoke Inspection, City of Cincinnati, was completed in January, 1952 (1). The instrument was regulated to expose a clean

one-inch diameter circle of filter paper every six hours. After a testing period, routine observations were recorded starting in the latter part of February.

The Bureau has established daily sampling periods as follows: 12 Midnight to 6 A.M. (Period A), 6 A.M. to 12 Noon (Period B), 12 Noon to 6 P.M. (Period C), and 6 P.M. to 12 Midnight (Period D). The staining effect of the sampled air appears in the form of a gray or black filter disc. Intensity of staining is evaluated by means of a "photo-volt" reflectance meter on which the per cent reflectance of clean paper is set at 100.

To make the stain index more closely proportional to the concentration of pollutants on the filter, the Bureau of Smoke Inspection converts observed reflectances to so-called "dirt shade" values according to the formula

$$\text{Dirt Shade} = \log \frac{\% \text{ reflectance clean } (= 100)}{\% \text{ reflectance observed}} \times (100) .$$

Finally, each dirt shade is adjusted to a number per 10,000 linear feet of air in order to allow comparisons among filters exposed to varying volumes of air during each six-hour period. This final index, defined as Reflectance Units of Dirt Shade (RUDS) is obtained by multiplying the dirt shade by a factor equal to the volume of air sampled during the period divided by the area of the sampling tube.

Temperature readings at various heights, from which the extent of inversion may be determined precisely, are not available in Cincinnati. However, qualitative estimates of inversion were recorded each morning

at the Bureau of Smoke Inspection. These observations have been scored as follows: 0 (no inversion), ½ (possible inversion), 1 (light inversion), 2 (heavy or definite inversion, smog).

Records relating to air pollution in Cincinnati during 1952 and 1953 have been made available through the kind cooperation of the Bureau of Smoke Inspection, City of Cincinnati.

C. Cardiovascular Mortality:

Mortality information for the year 1952 was obtained from mortality punch cards put at our disposal with the gracious cooperation of the Health Department of Cincinnati, Bureau of Vital Statistics. Only those deaths in Cincinnati of Cincinnati residents have been examined. This report is restricted to cardiovascular mortality, including the following categories: Diseases of the blood and blood-forming organs (International List Nos. 290-299), Vascular lesions affecting the central nervous system (Nos. 330-334), Arteriosclerotic and other heart disease (Nos. 420-434), Hypertensive disease (Nos. 440-447), Diseases of the arteries and veins (Nos. 450-468), Other diseases of the circulatory system (Nos. 400-416). Approximately 54 per cent of all deaths in Cincinnati during 1952 were attributed to these diseases.

Table I

Variable	Source	Type of Weekly Average	Polynomial Needed to Describe Seasonal Trend
1. Median Daily Temperature	City Office	Median	Quartic
2. Maximum Temperature	City Office	Weekly Maximum	Quartic
3. Minimum Temperature	City Office	Weekly Minimum	Quartic
4. Range (Max.-Min.)	City Office	Weekly Max. -Weekly Min.	Constant
5. Relative Humidity (%) (Daily 1 P.M. Readings)	Airport	Median	Quartic
6. % of Possible Sunshine	Abbe	Mean	Cubic
7. Daily precipitation (water equiv. in inches)	City Office	Total	Constant
8. Barometric Pressure (Daily 1 P.M. Readings)	Airport	Median	Cubic
9. Air Pollution: Staining index- RUDS (see text)	Bureau of Smoke Inspection City of Cincinnati	Mean for each period (see text)	Quadratic
10. Wind speed (M.P.H. during each of four daily periods)	as above	Mean for each period	*
11. Daily Inversion Score (see text)	as above	Mean	Cubic
12. Cardiovascular Mortality	Bureau of Vital Statistics, Health Department, City of Cincinnati	Total	Cubic

* Polynomial not fitted to this Variable. Wind speed correlated with RUDS and Inversion Score as described in text.

II. Methods

A. Testing the stain index as a measure of air pollution:

Since the depth of stain measurement is but one of many possible measures of atmospheric pollution, albeit the only one available to this study, its quality as an index of air pollution should be examined.

At best, RUDS are proportional to the concentration of very small particles of smoke and other colored particulates which may remain suspended in air for long periods of time and are able to stain objects on which they alight. Such particles form a large fraction of the total number of particles but an insignificant fraction of the total weight. It is possible that these small particles exert a disproportionately large detrimental effect on the human physiology by virtue of their ability to penetrate the protective lining of the lung. If this is true, the staining index may be a valuable measure of the health hazard existing in a given air pollution loading.

The relation of staining index to gases in the atmosphere is not well known. A study of the correlation between RUDS and the concentration of sulfur dioxide in the sampled air is currently being sponsored by the Bureau of Smoke Inspection.

The staining index may be tested indirectly by examining its correlation with meteorological factors known to affect air pollution. Primary among these are wind speed and temperature lapse rate, to both of which air pollution is inversely related.

As noted, no quantitative data on lapse rate is available in Cincinnati. Instead, a crude inversion score, based on early morning observations, has been derived as indicated above. The staining index, RUDS, should correlate directly with this score. Table II shows the linear correlation coefficients between wind speed, RUDS, and inversion score, based on measurements during 1952 at the site of the Bureau of Smoke Inspection. Since the inversion score relates to conditions in the early morning hours, it has been correlated with staining and wind speed during Periods A and B only.

Table II

Correlations Between Staining Index (RUDS), Inversion Score, and Wind Speed (Miles Per Hour), by Season and Period, 1952 *

Variables	Period							
	A		B		C		D	
	Fall[1]	Other Seasons[2]	Fall	Other Seasons	Fall	Other Seasons	Fall	Other Seasons
	Correlation Coefficients							
RUDS, Inversion	≠ .19	≠ .38	≠ .23	≠ .27				
RUDS, Wind	- .41	- .50	- .41	- .22	- .10	≠ .10	- .34	- .28
Inversion, Wind	- .46	- .33	- .33	- .34				

* Source: Bureau of Smoke Inspection, City of Cincinnati

[1] September 21 - December 31

[2] February 26 - September 20

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The fall season was segregated from the rest of the year because the staining index averaged about twice as high in this season as in the earlier months. Moreover, the frequency and extent of inversion, relative to the number of days of observation, were much greater in fall.

The number of paired observations ranged from 88 to 102 in fall and from 188 to 201 in the earlier months. Hence, fall correlation coefficients greater than $|.21|$ are significant at the 5 per cent level, and if greater than $|.27|$ are significant at the 1 per cent level. The corresponding critical points for the "other seasons" coefficients are $|.14|$ and $|.18|$, respectively. The more conservative 1 per cent points are safer guides since the distributions of RUDS and wind velocity were generally skewed to the left.

As a rule, correlations were strongest during Period A, the early morning hours. As noted below, fluctuations in RUDS during this period also correlated well with peaks and dips in cardiovascular mortality. The single poor correlation - between RUDS and inversion score in fall - resulted from several unusually high staining indices observed during mornings when no inversion was visible.

Correlations were negligible during Period C - afternoon. This time of day in Cincinnati is distinguished by a highly variable wind speed with frequent gusts and staining indices substantially less, on the average, than those in any other interval.

Inspection of variations in wind, RUDS, and inversion scores greater than zero revealed very poor correlation, indicating that the

attempt to discriminate between light and heavy inversions was not successful, or, at least, not necessary. In the absence of quantitative data on lapse rate, a scoring system of zero (no visible inversion) and unity (visible inversion) would suffice.

B. Correlating weather, air pollution, and mortality measurements:

This study hopes to detect the short-term effects, if any, of fluctuating environmental conditions on cardiovascular mortality. For purposes of analysis it is necessary to translate the word "short-term" into a specific interval of time. In this study, the week has been selected - a unit which represents a compromise between continuously varying environmental conditions and the need to restrict each variable to a manageable number of observations.

A week was defined as a Sunday to Saturday interval, the first week of 1952 beginning December 30, 1951, and the last ending January 3, 1953. Thus, 53 weeks of experience are included in this report.

A weekly average-in some cases, a total-of the daily observations provided a single datum to represent each variable. For continuous variables, such as temperature and barometric pressure, the median was used. For those variables which changed by discrete steps, the arithmetic mean appeared to be more representative of the situation during the week. Table I lists the type of average used for each variable.

It is believed that the best way to detect short-term effects is to correlate weekly deviations from the regular seasonal trend of an environmental variable with corresponding weekly deviations from the mortality trend. For example, we may test whether peaks (i.e.,

positive deviations from the seasonal trend) in air pollution during the fall weeks of 1952, or dips (negative deviations) in median temperature during the winter weeks, are associated with weekly peaks in deaths from cardiovascular diseases.

The first step was to fit to the weekly values for each variable a polynomial of degree sufficiently high to describe the general seasonal trend but not so high as to be affected by individual weekly fluctuations. The method of orthogonal polynomials is ideally suited to this task since curves of successively higher degree may be fitted with a minimum of additional computation. Further, a rapid test is available to determine whether the addition of each higher degree produces a significant decrease in residual variation.*

The mathematical theory is discussed by Kendall (2, pp. 145-152) and needs no elaboration here. Orthogonal polynomials for curves up to the fifth degree and for any number of equally spaced observations up to 75 are available in Fisher and Yates' Tables (3). Tables for sample sizes up to 104 observations have been prepared by Anderson and Houseman (4). The fact that such tables are restricted in their extent is sufficient practical reason for selecting the week rather than the day, for example, as the time unit on which to base a study of an entire year's experience. The type of polynomial required to describe the seasonal trend of each variable is given in the final column of Table I.

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* Frequently, the linear term is of no importance, whereas the graph of weekly values will show that a quadratic or cubic term is necessary. In this case, the curve-fitting process must be continued to obtain the coefficients of these terms.

The next step in analysis was to compute for each variable the set of fifty-three deviations between observed values and corresponding expected values given by the fitted polynomial. Following this, each set of environmental deviations was correlated separately with the set of mortality deviations in order to screen out those environmental factors of no apparent significance.

These correlations did not involve the numerical values of the mortality deviations, but rather their signs. Specifically, a deviation from the fitted mortality trend was assigned a value of unity if positive, zero if negative. This procedure was followed preliminary to the determination of a linear "discriminant" function, that is, a mathematical expression of the form

$$z = a_0 + a_1 x_1 + a_2 x_2 + \cdots + a_n x_n . \quad (1)$$

where the x's represent environmental variables whose fluctuations appear to be significant predictors of a weekly rise or fall in mortality, while the a's are relative weights to be assigned those variables. The object is to provide an index, namely, the value of z, whereby the occurrence of a peak or dip in cardiovascular mortality (but not its magnitude) may be predicted with some reasonable degree of confidence. The computation and application of such a function are discussed briefly below.

In order that the correlation statistic, r, known as a point biserial coefficient - may serve in the selection of useful environmental predictors, a statistical significance test is required. This has been provided in a recent note by Tate, (5).

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In small samples of size n, the statistic

$$T = r\sqrt{(n-2)/(1-r^2)}$$

is distributed in Student's "t" distribution with n-2 degrees of freedom, under the null hypothesis that the true correlation is zero. Hence, we reject this hypothesis if $|T|$ assumes a value greater than $|t_{\alpha, n-2}|$, where α is the chosen level of significance. Since the power of this test to detect a true correlation different from zero will not be great for small samples, it is unwise to choose too small a value of α. A ten per cent significance level seemed reasonable in the present study.

It soon became apparent that certain variables (e.g., median temperature) correlated positively with mortality in the summer but negatively in the winter. This situation was also noted by Reed (6) in 1924. In addition, some variables showed significant correlation with mortality during one season but not another.

Consequently, the year was divided arbitrarily into three periods: the winter quarter, including the first twelve weeks of study, ending March 22; the spring and summer half, beginning March 23 and ending September 20 and the fall quarter comprising the remaining weeks. Within the spring and summer half, the hot, midsummer period from May 25 to August 9, appeared to offer some particularly useful information and was subjected to separate study.

Within each period, the weekly mortality scores (zero or unity) were correlated with the environmental deviations during the preceding as well as the current week. This led to correlation

coefficients of lag one and lag zero, respectively. The former tests the existence of a time lag between the appearance of an environmental fluctuation and its apparent effect on the death rate from cardiovascular disease.

Those environmental variables yielding significant lag zero or lag one correlations are included in Table III. The last column of this table gives the critical values of the coefficient at the ten per cent significance level, assuming a true correlation of zero.

Table III

Time Period	Variable	Observed Correlation with Cardiovascular Mortality Deviations	Correlation Just Significant at 10 Per Cent Level
Winter Quarter (12 observations)			
Lag zero:	Median Temperature	- .53	| .50 |
	Relative Humidity	- .56	
	% Possible Sunshine	- .66	
Lag one*:	Precipitation	≠ .52	| .53 | *
	Barometric Pressure	- .70	
Spring and Summer (26 observations)			
Lag one:	Median Temperature	≠ .38	| .33 |
	Maximum "	≠ .49	
	% Possible Sunshine	≠ .37	
	R.U.D.S.: Period A	≠ .51	
	" : Period B	≠ .36	

* These correlations based on eleven rather than twelve observations since the environmental deviation needed to match the mortality deviation for the first week of 1952 would have had to be obtained from data during the last week of 1951. A further footnote on this problem in connection with discriminant function analysis follows below.

Table III (Cont'd.)

Time Period	Variable	Observed Correlation with Cardiovascular Mortality Deviations	Correlation Just Significant at 10 Per Cent Level
Weeks #22-32 (May 25-Aug. 9) (11 observations)			
Lag zero:	Temperature Range	- .59	.53
	Precipitation	≠ .57	
	Barometric Pressure	≠ .74	
Lag one:	R.U.D.S.: Period A	≠ .59	
Fall Quarter (15 observations)			
Lag zero:	Median Temperature	- .45	.45
Lag one:	R.U.D.S.: Period A	≠ .57	
	" : Period B	≠ .53	

It is noteworthy that fluctuations in RUDS observed in Period A were positively correlated with peaks and dips in cardiovascular mortality during every season offering sufficient data for analysis.* Moreover, the correlation was consistently with mortality during the subsequent week - an interesting lag association. In contrast, inversion showed no correlation with peaks in mortality.

Barometric pressure and mortality appear to be correlated inversely in winter but directly during the extreme heat of June and July. This change in the sign of the relationship is not surprising when the general meteorology of the Cincinnati area is considered. Low pressure area in winter are known to precede stormy,

* Only four weeks of routine sampling with the automatic smoke filter had been completed before March 20, the end of the winter quarter. Therefore, no correlation analysis between RUDS and mortality was attempted for this quarter.

cold weather, while stable highs in summer are associated with continued oppressive heat and a narrow temperature range.

C. Discriminant Function Analysis:

Not all the variables listed in Table III were selected for further analysis. For example, barometric pressure was used, but not precipitation. These two tend to be inversely related, but the former showed a stronger correlation with mortality and, moreover, is uniform over a larger area at a given time.

Variables chosen for discriminant function analysis, by season, were:

1. Winter quarter

   Lag zero - median temperature, relative humidity, per cent possible sunshine

   Lag one - barometric pressure*

2. Spring and summer half

   Lag one - maximum temperature, RUDS (Period A)

3. May 5 - August 9

   Lag zero - temperature range, barometric pressure

   Lag one - RUDS (Period A)

4. Fall Quarter

   Lag zero - median temperature

   Lag one - RUDS (Period A)

The mathematics involved in computing the relative weights in the discriminant function (i.e., the a's in Eq. (1) is described by Hoel (7, pp. 121-126) and - on a more advanced level - by Rao (8, pp. 287 ff.). These weights are determined by inverting the

* The deviation in barometric pressure during the last week of 1951 was estimated by regressing the 1952 polynomial to cover this week.

matrix of variances and covariances of the variables. A clear, step-by-step procedure for computing a reciprocal matrix has been given by Anderson and Bancroft (9, pp. 197-200).

After the weights had been obtained, a value of $\underline{z}$ for each week was calculated from Eq. (1), the x's representing weekly deviations of the selected variables from their respective trend curves. If these variables are, in fact, good predictors of peaks and dips in cardiovascular mortality, then the difference between the mean z's for positive and negative mortality deviations will be large relative to the variation in individual z's within each group of mortality deviations. Specifically, z's associated with positive mortality deviations should be positive and their mean a large positive number, while those associated with negative mortality deviations should be negative and their mean a large negative number.

This ideal will not be fully realized if the biserial correlations between the environmental variables and the zero-one mortality scores are substantially less than unity. In any case, the effectiveness of the discriminant function may be examined by an analysis of variance in which the difference between the two mean z's is compared to the variation within each group by means of an F-test.

Table IV lists, by season, the relative weights computed for each variable, the mean z's, and the F-values obtained from analyses of variance. Following each F, in parentheses, are the degrees of freedom "between" and "within" the two groups of z's. The former is equal to the number of environmental variables used in the discriminant

analysis, the latter to one less than the total number of z's in each season minus the number of variables.

Table IV

Summary of Discriminant Function Analysis

Season	Variable	Relative Weights(a's)	Mean z's Pos.Mort. Dev's.	Mean z's Neg.Mort. Dev's.	F
Winter	Med.Temp. (lag zero)	- 2.3	33.3	- 49.8	4.4(4,7)
	Rel. Hum. (" ")	- 1.0			
	%Poss.Sunsh.(" ")	- 3.0			
	Bar.Press. (" one)	- 161.5			
Spring and Summer	Max.Temp. (lag one)	1.0	4.7	- 3.7	9.5(2,23)
	RUDS:Per.A(" ")	1.0			
May 25-August 9	Temp.Range(lag zero)	- 1.7	17.6	- 13.3	9.2(3,7)
	Bar.Press.(" ")	142.9			
	RUDS:Per.A(" one)	1.0			
Fall	Med.Temp. (lag zero)	- 1.2	9.0	- 4.9	4.4(2,12)
	RUDS:Per.A(" one)	1.0			

Each weight in Table IV bears the same sign as the correlation coefficient pertaining to that variable (see Table III). The numerical value of the weight is directly related to the strength of the correlation and inversely to the general magnitude of deviations of the variable from the fitted trend curve. For example, both these factors (especially the latter) operate to yield an extremely heavy weight for barometric pressure.

The values of F given in Table IV are all significant at the five per cent level; those for the spring and summer half and the midsummer period are significant at the one per cent level as well.

We may conclude that the selected environmental variables possessed significant but not exceptional power to predict whether the number of cardiovascular deaths in a given week of 1952 would be above or below that expected from the general year-round trend.

To apply these results in practice, it would be necessary to follow - by inspection, at least, - the general trends in the selected environmental variables for a number of weeks previous to the current one. Deviations from these trends during the current or preceding week, depending on the variable, would then be estimated, relative weights applied, and a value of z computed. If this value were less than the midpoint of the two mean z's given in Table IV for the current season, a decline in cardiovascular mortality would be indicated; if the calculated z were greater than this midpoint, a rise in mortality may be predicted.

Obviously, the specific results obtained from 1952 data may be modified or perhaps contradicted by studies of Cincinnati experience during other years (see below). Similar studies in other cities may produce entirely different results. It is the writer's opinion that the methodology outlined in this report is more important than the specific results.

One further set of calculations should be mentioned. Analyses so far reported have involved the sign of the mortality deviation but not its magnitude. It is of interest to determine whether weekly deviations in one or more environmental variables are correlated with the extent as well as direction of mortality changes.

Attention was restricted to positive deviations (i.e., peaks) in cardiovascular deaths. Within each season, these deviations were arrayed in order of decreasing size and matched with deviations in selected environmental variables (during current or preceding week as indicated by prior analyses). The latter included not only those chosen for discriminant function analyses but also several others in each season. They were then examined to detect the existence of any relationship to the declining mortality deviations.

Results were generally discouraging. In fall, for example, there were two weeks with almost identical mortality deviations, both substantial - about ten deaths higher than expected from the year-round trend curve. For one of these weeks, the median temperature (lag zero) was 2.3 degrees below expected while the staining index (lag one) was 2.2 units above its trend curve. For the other week, however, the median temperature was 10.8 degrees below expected while the appropriate staining index was 17.5 units above. Deviations in relative humidity and per cent possible sunshine (lag zero) also differed markedly from one week to the other.

A more instructive example was offered by Week 27 (June 29 - July 5). This interval provided by far the largest positive deviation in cardiovascular mortality of any week in 1952: thirty-five deaths above the trend curve. Yet the maximum temperature for this week was only 1.3 degrees above expectation, while the previous week's staining index was slightly below its expected value. On the other hand, maximum temperature had been higher and temperature

range narrower than expected for three to four weeks in succession immediately prior to Week 27. Perhaps the unusually high death rate during this week represented an effect which was building up during earlier weeks but needed the excitement of the July 4th holiday to trigger it into being.

In summary, the direction of a deviation in weekly deaths from cardiovascular disease may be predicted fairly well from certain environmental variables studied here, but the magnitude of the change is of too complex an origin to be accounted for solely in terms of these variables.

Studies Now in Progress:

Analyses identical with those described in this report are being undertaken on Cincinnati data of 1953 (January 4, 1953 - January 2, 1954). Negotiation of a contract with the I.B.M. Service Bureau in the city for the computation of polynomial trend curves and deviations, using the electronic calculator, has speeded the work considerably. Effects of environmental fluctuations on respiratory (excluding tuberculosis and cancer of the lung) as well as cardiovascular deaths during 1952 and 1953 are under study. It is hoped that these studies will be completed by September of this year.

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